

Polynomial Division

Example

Factor $x^2 - 5x + 6$

Example

Factor $x^3 - 3x + 2$

Example

If $4x^3 + 2x^2 + 3 \equiv (x - 2)(Ax^2 + Bx + C) + R$, find A , B , C and R

Theorem (Polynomial Division)

If we have polynomials $f(x)$ and $g(x)$ of **degree** n and m , then we can write $f(x) = g(x)q(x) + r(x)$ where $q(x)$ has degree $n - m$ and $r(x)$ has degree less than m .

Factor Theorem

Theorem (Factor Theorem)

For a polynomial $f(x)$, $f(a) = 0 \iff (x - a)$ is a factor of $f(x)$

Tip

Make sure you're using the correct direction!

Theorem (Remainder Theorem)

For a polynomial $f(x)$, $f(a) = r \iff f(x) = (x - a)q(x) + r$ for some polynomial $q(x)$. Said another way, when $f(x)$ is divided by $x - a$ the remainder is $f(a)$.

Tip

We've not said whether or not the roots are integers, rationals or reals. One useful form is sometimes, if $f\left(\frac{p}{q}\right) = 0 \iff f(x) = (qx - p)g(x)$

Example

Factorise *completely* $x^4 - 3x^3 + 4x^2 - 8$

Example

Solve the equation

$$(x^2 - 3x + 3)^2 - 3(x^2 - 3x + 3) + 3 = x$$